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Quantum Tunneling of Dirac Particles from the Generalized Spherical Symmetric Evaporating Charged Black Hole

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Abstract Kerner and Mann's recent research shows that, for an uncharged and non-rotating black hole, its Hawking temperature and tunneling rate can be exactly obtained by the fermion tunneling method from its event horizon. In this paper, considering the tunneling charged particles with spin 1/2, we extend Kerner and Mann's method to the generalized spherical symmetric evaporating charged black hole which is non-stationary. In order to investigate the fermion tunneling through the event horizon, we choose a set of appropriate matrices γ^{μ}. As a result, the tunneling probability and truly effective temperature are well recovered by charged fermions tunneling from the black hole.

Keywords Quantum tunneling · Dirac particles · Evaporating charged black hole · Effective temperature

1 Introduction

The research on radiation effect of black hole has been one of the most important contents in both theoretical physics and astrophysics since thermal radiation of black hole was firstly discovered and proved by Hawking in the 1970's [1, 2]. A considerable amount of work has been done relating to various black holes' thermodynamic properties in the past few decades [3–7]. In 2000, Parikh and Wilzcek's work, which treats the Hawking radiation as semi-classical tunneling process from the event horizon of static Schwarzshild and Reissner-Nordström black holes, indicates that the factually radiant spectrum is related to the change of Bekenstein-Hawking entropy, and deviates from the precisely thermal spectrum after taking the self-gravitation interaction and energy conservation into account [8, 9].

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Furthermore, Zhang and Zhao have extended this method from spherically symmetric black holes to general axisymmetric black holes [10, 11], and even to the cases of the massive and charged particle's tunneling [12, 13]. Since then, making use of these tunneling methods, a lot of people have investigated Hawking radiation as tunneling from various black holes [14–23]. But most papers have only considered the scalar particle's tunneling radiation. In fact, a black hole can radiate any types of particles, and the true emission spectrum should contain the contributions of particles with spin.

Very recently, Kerner and Mann have studied the Hawking radiation of spin 1/2 particles of the spherically symmetric uncharged black hole and recovered the Hawking temperature of the black hole by fermion tunneling [24–26]. This method has been mentioned in recent studies [27–33], but for the case of non-stationary black hole, the characteristics of fermion tunneling have not been studied deeply. As we known, due to absorption and evaporation, the mass and charge of black holes are not fixed, and stationary black holes are only ideal models. So the study of non-stationary black hole is rather meaningful in the cognition and exploration on black holes.

In this paper, we extend Kerner and Mann's method to the generalized spherical symmetric evaporating charged black hole which is non-stationary. We choose a set of appropriate matrices γ^{μ} for general covariant Dirac equation. This is an important technique for fermions tunneling method. As a result, the tunneling probability and truly effective temperature are well recovered by charged fermions tunneling from the black hole.

2 The Event Horizon of the Black Hole

The line element of the generalized spherical symmetric evaporating charged black hole represented in Eddington-Finkelstein coordinates can be expressed as

$$\begin{aligned} ds^2 = {} & -e^{2\psi}\left(1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}\right)dv^2+2e^{\psi}\,dv dr \\ & + r^2(d\theta^2+\sin^2\theta\, d\varphi^2), \end{aligned} \tag{1}$$

with the electromagnetic potential

$$A_\mu=\left(-\frac{Q(v,r)}{r},0,0,0\right), \tag{2}$$

where $\psi=\psi(r,v)$. M and Q are the mass and charge of the black hole, respectively. This line element is different from that of a stationary sphere-symmetrically charged black hole, and the mass M and charge Q of the black hole vary with time. The components of non-null inverse metric tensor are represented as

$$\begin{aligned} & g^{vr}=g^{rv}=1, \qquad g^{rr}=e^{\psi}\left(1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}-2\dot{r}_H e^{-\psi}\right), \\ & g^{\theta\theta}=e^{\psi}r^{-2}, \qquad g^{\varphi\varphi}=r^{-2}\sin^{-2}\theta e^{\psi}. \end{aligned} \tag{3}$$

According to the null hyper-surface equation

$$g^{\mu\nu}\frac{\partial f}{\partial x^\mu}\frac{\partial f}{\partial x^\nu}=0, \tag{4}$$

we can get the event horizon equation of the black hole

$$(1 - 2\dot{r}_H e^{-\psi(r_H,v)}) r_H^2 - 2Mr_H + Q^2 = 0. \tag{5}$$

Therefore the position of event horizon is

$$r_H = \frac{M + \sqrt{M^2 - Q^2(1 - 2\dot{r}_H e^{-\psi})}}{1 - 2\dot{r}_H e^{-\psi}}, \tag{6}$$

where $\dot{r}_H = \frac{dr_H}{dv}$. From metric (1), we find this coordinate system has many good properties, such as no singularity at the event horizon and the complete converge of the whole space-time manifold.

3 Tunneling Probability and Effective Temperature at the Event Horizon

In this section, based on Kerner and Mann's research, we focus on studying Hawking radiation of fermion via tunneling from generalized spherical symmetric evaporating charged black hole at the event horizon. The Dirac equation with electric charge is

$$\gamma^\mu \left(\partial_\mu + \frac{i}{2} \Gamma_\mu^{\alpha\beta} \Sigma_{\alpha\beta} + \frac{iq}{\hbar} A_\mu \right) \Psi + \frac{m}{\hbar} \Psi = 0, \tag{7}$$

where

$$\Gamma_\mu^{\alpha\beta} = g^{\beta\gamma} \Gamma_{\mu\gamma}^\alpha, \qquad \Sigma_{\alpha\beta} = \frac{i}{4} \left[\gamma^\alpha, \gamma^\beta \right], \qquad \left\{ \gamma^\alpha, \gamma^\beta \right\} = 2g^{\alpha\beta} \times I. \tag{8}$$

To deal with Hawking radiation of fermions, we should first choose an appropriate γ^μ matrices. For simplicity, γ^μ matrices can be chosen as

$$\begin{aligned}
\gamma^V &= \frac{e^\psi}{\sqrt{1 - \frac{2M(v,r)}{r} + \frac{Q^2(v,r)}{r^2}}} \begin{pmatrix} -i & \sigma^3 \\ \sigma^3 & i \end{pmatrix}, \\
\gamma^R &= \sqrt{1 - \frac{2M(v,r)}{r} + \frac{Q^2(v,r)}{r^2}} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \\
\gamma^\theta &= \frac{1}{r} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \\
\gamma^\varphi &= \frac{1}{r \sin\theta} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},
\end{aligned} \tag{9}$$

where σ^i $(i = 1, 2, 3)$ are Pauli matrices as follows:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{10}$$

and we denote $\xi_{\uparrow}/_{\downarrow}$ for the eigenvectors of σ^3. For 1/2 spin fermions, the wave functions have two spin states, and we employ the following ansatz as

$$\Psi_\uparrow(V, R, \theta, \varphi) = \begin{pmatrix} F(V, R, \theta, \varphi)\xi_\uparrow \\ H(V, R, \theta, \varphi)\xi_\uparrow \end{pmatrix} \exp\left(\frac{i}{\hbar} I_\uparrow(V, R, \theta, \varphi) \right)$$

$$= \begin{pmatrix} F(V,R,\theta,\varphi) \\ 0 \\ H(V,R,\theta,\varphi) \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar} I_\uparrow(V,R,\theta,\varphi)\right). \tag{11}$$

Inserting the ansatz (11) for spin up particles into the Dirac equation (7), thus, we get the following equations

$$\frac{1}{\sqrt{1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}}}(iF-H)\left(e^{-\psi}\partial_V I_\uparrow + \frac{Q(v,r)q}{r}\right) - H\sqrt{1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}}\,\partial_r I_\uparrow + mF = 0, \tag{12}$$

$$-\frac{H}{r}\left(\partial_\theta I_\uparrow + \frac{i}{\sin\theta}\partial\varphi I_\uparrow\right) = 0, \tag{13}$$

$$-\frac{1}{\sqrt{1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}}}(F+iH)\left(e^{-\psi}\partial_V I_\uparrow + \frac{Q(v,r)q}{r}\right) - F\sqrt{1-\frac{2M(v,r)}{r}+\frac{Q^2(v,r)}{r^2}}\,\partial_r I_\uparrow + mH = 0, \tag{14}$$

$$-\frac{F}{r}\left(\partial_\theta I_\uparrow + \frac{i}{\sin\theta}\partial\varphi I_\uparrow\right) = 0. \tag{15}$$

It is difficult to directly get the expression of the action. Considering the properties of the black hole, we carry out the separation of variables as

$$I_\uparrow(V,r,\theta,\varphi) = -\int e^{\psi(v,r)}\omega\, dV + W(r) + Y(\theta,\varphi), \tag{16}$$

where ω is the energy of the emitted particle measured by an observer at infinity. Inserting (16) into (12–15), we have

$$\frac{-iF}{\sqrt{\mathcal{N}(r,v)}}\left(\omega - \frac{Q(v,r)q}{r}\right) + \frac{H}{\sqrt{\mathcal{N}(r,v)}}\left(\omega - \frac{Q(v,r)q}{r}\right) - H\sqrt{\mathcal{N}(r,v)}\,W'(r) + mF = 0, \tag{17}$$

$$-\frac{H}{r}\left(Y_\theta(\theta,\varphi) + \frac{i}{\sin\theta}Y_\varphi(\theta,\varphi)\right) = 0, \tag{18}$$

$$\frac{F}{\sqrt{\mathcal{N}(r,v)}}\left(\omega - \frac{Q(v,r)q}{r}\right) + \frac{iH}{\sqrt{\mathcal{N}(r,v)}}\left(\omega - \frac{Q(v,r)q}{r}\right) - F\sqrt{\mathcal{N}(r,v)}\,W'(r) + mH = 0, \tag{19}$$

$$-\frac{F}{r}\left(Y_\theta(\theta,\varphi) + \frac{i}{\sin\theta}Y_\varphi(\theta,\varphi)\right) = 0, \tag{20}$$

where

$$\mathcal{N}(r,v) = 1 - \frac{2M(v,r)}{r} + \frac{Q^2(v,r)}{r^2}. \tag{21}$$

Equations (18) and (20) imply that $Y(\theta, \varphi)$ is a complex function. The solution of $Y(\theta, \varphi)$ is the same as the spin-down case, so its contribution to the rate emission cancels out. Note that, whatever $m = 0$ or $m \neq 0$, it does not affect the result. As regard the remaining equation, when $F = iH$, we have

$$W_+(r) = \int \frac{2(\omega - \frac{Q(v,r)q}{r})}{\mathcal{N}(r, v)} dr = \int \frac{2(\omega - \frac{Q(v,r)q}{r})}{\mathcal{N}'(r, v)(r - r_H)} dr$$
$$= \frac{(\omega - \frac{Q(r_H,v)q}{r_H}) r_H^3 \pi i}{r_H M(r_H, v) - Q^2(r_H, v)}, \tag{22}$$

where $\mathcal{N}'(v, r) = \frac{\partial \mathcal{N}(v,r)}{\partial r}$, and when $F = -iH$, then

$$W'_-(r) = 0, \tag{23}$$

which corresponds to the incoming particle absorbed in the classical limit with probability $P(\text{absorption}) = 1$. From (22), we can get the imaginary part of the act

$$\operatorname{Im} I = \operatorname{Im} W_+(r)$$
$$= \frac{\pi r_H^3}{r_H M(r_H, v) - Q^2(r_H, v)} (\omega - \omega_0), \tag{24}$$

where $\omega_0 = \frac{Q(v,r_H)q}{r_H}$. The above integral is divergent at $R = 0$, which corresponds to the horizon $r = r_H$. Making use of a prescription corresponding to the Feynman propagator, we can get the non-vanishing imaginary part of the action. As a result, the tunneling probability of the fermion is

$$\Gamma = \frac{P(\text{emission})}{P(\text{absorption})} = \exp(-2 \operatorname{Im} W_+(r))$$
$$= \exp\left(\frac{-2\pi r_H^3}{r_H M(r_H, v) - Q^2(r_H, v)} (\omega - \omega_0)\right), \tag{25}$$

and effective temperature of the black hole is

$$T = \frac{\mathcal{N}'(r_H, v)}{4\pi} = \frac{r_H M(r_H, v) - Q^2(r_H, v)}{2\pi r_H^3}. \tag{26}$$

This confirms that the fermions tunneling method is still working and is valid for such non-stationary charged black hole.

4 Conclusion

In this paper, through choosing a set of appropriate matrices γ^μ, we successfully recover the effective temperature of the non-static black hole in tunneling process. Because the generalized spherical symmetric evaporating charged black hole is non-stationary and the mass M and charge Q of the black hole change with time, tunneling probability of the fermion and effective temperature are not fixed, and also vary with time. For the generalized spherical symmetric evaporating charged black hole, since it is spherical symmetry and has no angular momentum for itself, there is no coupling effect between the spin fermions and the angular momentum of the black hole in the tunneling rate. We hope fermions tunneling from non-stationary black hole with one or more angular momentum, the spin coupling effect will be presented. This is also an open question.

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